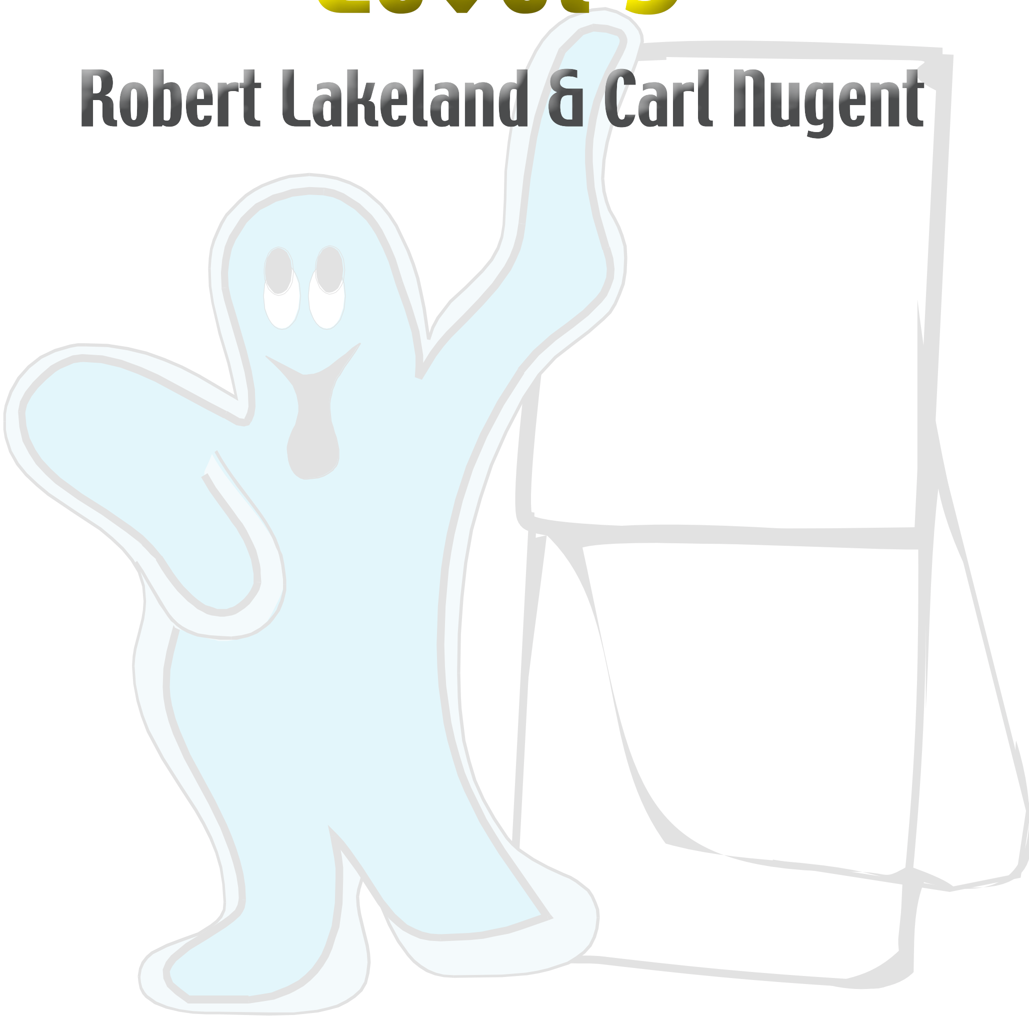
Year 10

Mathematics Workbook

Level 5

Robert Lakeland & Carl Nugent



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Contents

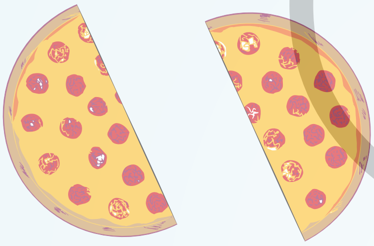
1.0 Number

1.1 Fractions

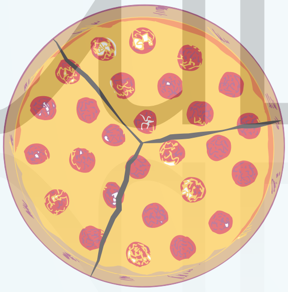
Review of Fractions

We already have a good understanding of fractions. We know the top line is called the numerator and the bottom line is called the denominator.

We also know that when we are getting half $\left(\frac{1}{2}\right)$ a pizza that the pizza will be cut evenly into two.

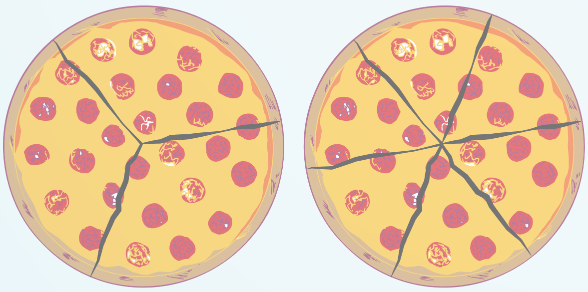


Similarly if there are three people sharing the pizza then each will have a third $\left(\frac{1}{3}\right)$.



When the pizza is delivered precut into six pieces we know each of the three people will get two slices as

$$\frac{2}{6} = \frac{1}{3}.$$



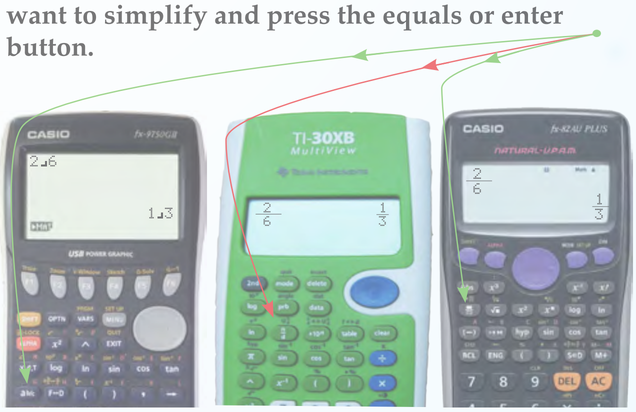
One third of a pizza is the same as two sixths of a pizza.

Extra Notes

In previous years manipulation of fractions has been taught both by written methods and using calculators. The approach in this workbook is to use calculators where possible but still to refer to the written method, so students understand and can calculate answers.

You can use the fraction button on your calculator to quickly simplify equivalent fractions.

Each calculator is a little different so check out your calculator now. Just enter the fraction you want to simplify and press the equals or enter button.



Achievement – Convert the following fractions to decimals.

87. $\frac{4}{5}$ 88. $\frac{5}{8}$ 89. $\frac{7}{20}$ 90. $\frac{11}{40}$

91. $3\frac{3}{5}$ 92. $9\frac{4}{5}$ 93. $2\frac{9}{40}$ 94. $11\frac{1}{8}$

Convert the following decimals to fractions.

95. 0.75 96. 0.45 97. 0.7 98. 0.55

99. 0.125 100. 3.65 101. 0.0625 102. 5.05

Convert the following fractions to approximate decimals, round off your answer to four decimal places.

103. $\frac{5}{12}$ 104. $\frac{2}{9}$ 105. $3\frac{2}{3}$ 106. $5\frac{4}{7}$

Example

Convert each fraction to a decimal and use the decimal answers to decide which of these fractions is larger.

$$\frac{6}{10} \text{ and } \frac{5}{8}.$$

We need each as a decimal.

$$\frac{6}{10} = 0.6$$

$$\frac{5}{8} = 0.625$$

Therefore $\frac{5}{8}$ is bigger than $\frac{6}{10}$.

Merit – Convert each pair of fractions to decimal equivalence to decide which fraction is larger.

107. $\frac{3}{4}$ and $\frac{7}{9}$

108. $3\frac{3}{5}$ and $3\frac{11}{20}$

109. $\frac{31}{40}$ and $\frac{4}{5}$

110. $\frac{1}{3}$ and $\frac{7}{20}$

Example

In a basket of Braeburn apples, seven out of forty apples were rotten. What percentage were rotten?

We write our expression as a fraction and multiply by 100 to make it a percentage.

7 out of 40 is $\frac{7}{40} = \frac{7}{40} \times \frac{100}{1}\%$

$= 17\frac{1}{2}\%$

If we wanted this as a decimal we would use the fraction convert button F↔D on the calculator.

Example

In a mathematics class there are thirteen girls and eleven boys. What percentage of the class are boys? Round the answer to one decimal place.

We write our expression as a fraction of the 24 in the class and multiply by 100 to make it a percentage.

11 out of 24 is $\frac{11}{24} = \frac{11}{24} \times \frac{100}{1}\%$

$= 45.8\%$

Achievement – Express the answer to each problem as a percentage. Round percentage answers to 1 decimal place.

121. The school soccer team has won twelve of the twenty games played this year. What percentage has it won?

122. In a spelling competition Jacob got 35 of the 40 words correct. What percentage did Jacob get right?

123. A PE class were shooting from the free throw line in basketball. Each of the twenty five students had three shots for a total of 16 baskets. What percentage of shots went in?

124. Of the 320 students in Year 9, 190 walk to school, 40 come by bus, 20 are dropped off by their parents and the remaining students bike to school. What percentage bike to school?

125. A shoe shop offered to discount the price by $10 a pair. The cost of a pair of shoes is $60. What is the percentage discount offered?

126. Students in Year 13 must attend 80% of lessons to graduate. Nicki has attended 100 out of 140 lessons. Calculate Nicki's attendance percentage.

127. Grace earns $15 an hour. On her next birthday her wages will increase by 50¢ an hour. What percentage is this increase?

128. Petra was told that 20% of New Zealanders smoke. She has asked all thirty teachers at her school and only four smoke. What percentage of teachers smoke?

Percentage Change

It is common to show the effect of a price change as a percentage as any change is likely to have a proportionate effect on all the items. Expensive items will increase by a larger dollar amount than a cheap item but usually by the same percentage.
To increase or decrease an amount by a percentage we could calculate the percentage and add or subtract it. Demonstrating this with a 15% increase for \$64.20

$$15\% \text{ of } \$64.20 = 64.20 \times \frac{15}{100}$$

$$15\% \text{ of } \$64.20 = \$9.63$$

$$\text{New Price} = 64.20 + \$9.63$$

$$= \$73.83$$

Alternatively if we are increasing the price by 15% then the new price is 115% of the old price.

$$\text{New price} = 64.20 \times 115\%$$

$$\text{New price} = 64.20 \times \frac{115}{100}$$

$$= \$73.83$$

Adding (or subtracting) the percentage to 100 means we can calculate the final figure in one step.

Extra Notes

Example

A restaurant decided to increase all prices by 8%. Find the new prices for each desert item on this menu.

Solution

We need 100+8 = 108% of each item.

Apple & Blueberry Pie

$$108\% \text{ of } 4.99 = \$4.99 \times \frac{108}{100}$$

$$= \$5.39$$

Cheesecake

$$108\% \text{ of } 5.99 = \$5.99 \times \frac{108}{100}$$

$$= \$6.47$$

Chocolate Cake

$$108\% \text{ of } 3.99 = \$3.99 \times \frac{108}{100}$$

$$= \$4.31$$

All answers rounded to the nearest cent.

Example

Toby has worked in the local service station for two years and he is now entitled to a 6% pay rise.

At present he gets \$15.30 an hour, what is his new pay rate to the nearest cent?

Solution

We know 100% is \$15.30 so we need to find 106%.

$$100\% \text{ is } = \$15.30$$

$$106\% \text{ is } = \$15.30 \times \frac{106}{100}$$

$$106\% \text{ is } = \$16.218$$

$$= \$16.22$$

He will now earn \$16.22 an hour.

2.0 Algebra

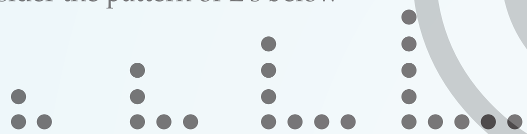
2.1 Patterns and Relationships – Review of Level 4

Patterns and Relationships

In Level 4 our focus was on linear patterns and relationships but in Level 5 we will extend this to include quadratic relationships.

From last year you should remember that a mathematical pattern is a relationship that follows a rule and that we can represent a pattern graphically, in a table or by a rule or equation.

Consider the pattern of L's below



Pattern 1 Pattern 2 Pattern 3 Pattern 4

Representing the pattern number and number of dots in each L design we get the following:

Pattern	Dots
1	3
2	5
3	7
4	9

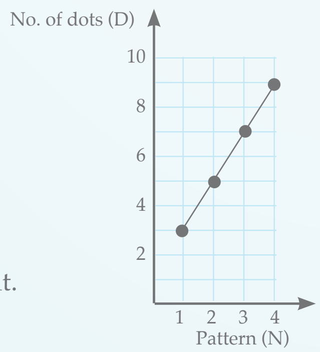
If we take a closer look at the table of values (dots), specifically the difference between successive dots, called the first differences, they are constant (the same).

Pattern	Dots	First Differences
1	3	
		2 ← 5 – 3 = 2
2	5	
		2 ← 7 – 5 = 2
3	7	
		2 ← 9 – 7 = 2
4	9	

This means that the pattern or relationship is a linear one.

Remember a linear relationship is one that when drawn on a set of axes gives a straight line.

See the graph on the right.



Extra Notes

Another approach to find the amount we add or subtract is to find the term before the first term.

For the sequence 8, 11, 14, 17, the term before the first term 8 is 5. Since the difference is +3 between each term the rule is D = 3n+ 5.

The rule that gives the number of dots for a specific L pattern is:

Multiply the pattern number by 2 and add 1 to get the number of dots in each L pattern.

Representing the L pattern as a rule or equation we get

$D = 2P + 1$, where D is the number of dots and P is the pattern number.

To find the rule for the pattern we can use the constant from our table of first differences as the starting point. Because the constant difference is 2, it means our rule involves multiplying each pattern number by 2.

To find out the amount we have to add or subtract after multiplying by 2, to get the correct pattern value, we try one or two terms.

For pattern 1, 2 x 1 = 2, but we require 3 so we need to add 1.

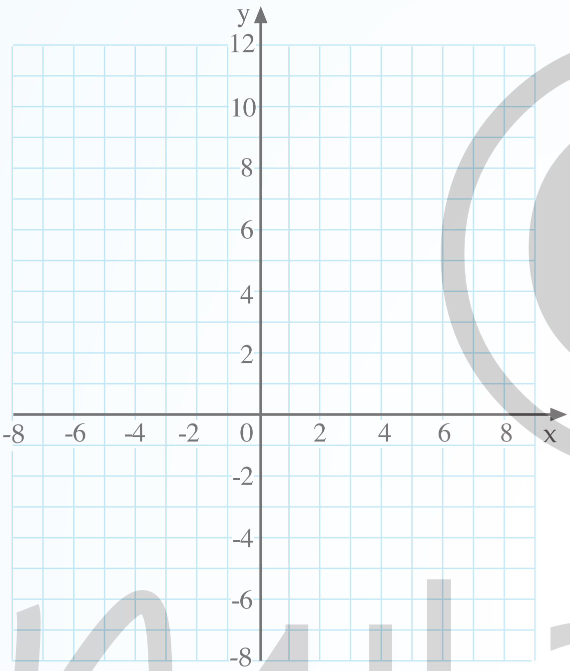
For pattern 2, 2 x 2 = 4, but we require 5 so we need to add 1.

So the rule that describes or generates this pattern is $D = 2P + 1$, where D is the number of dots and P is the pattern number.

Achievement – Graph each of the following lines using the two point method. A three row table has been provided for each graph in order for you to calculate your two points and check point.

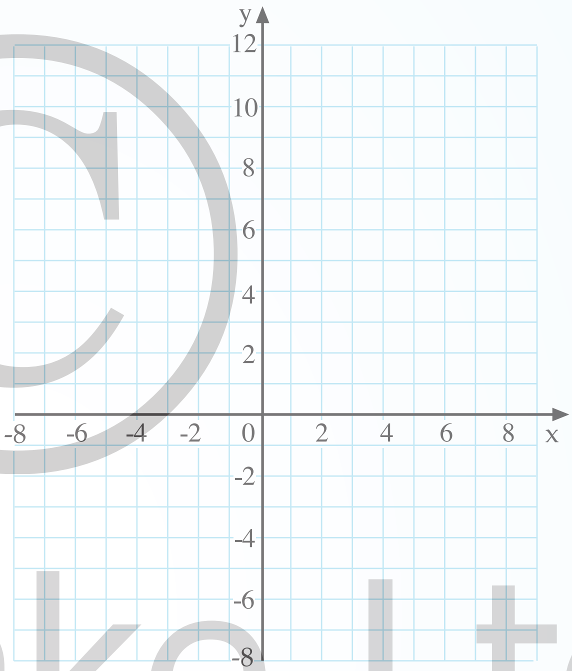
18. y = 2x + 5

	x	y
Point 1		
Point 2		
Check		



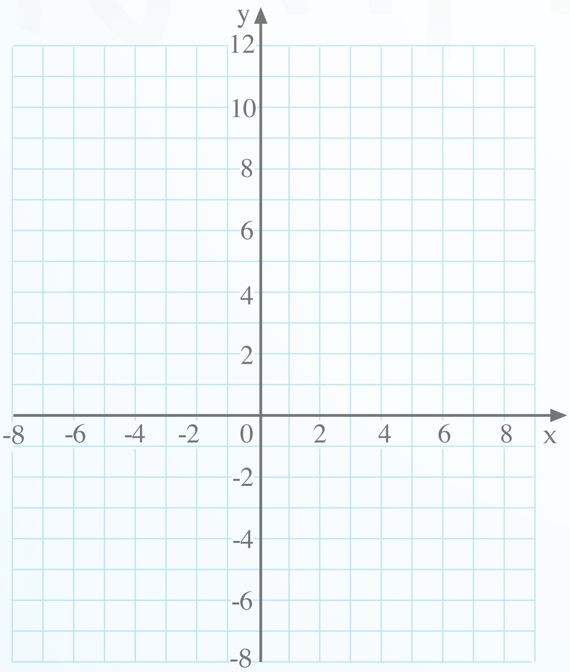
19. y = ⁻3x – 2

	x	y
Point 1		
Point 2		
Check		



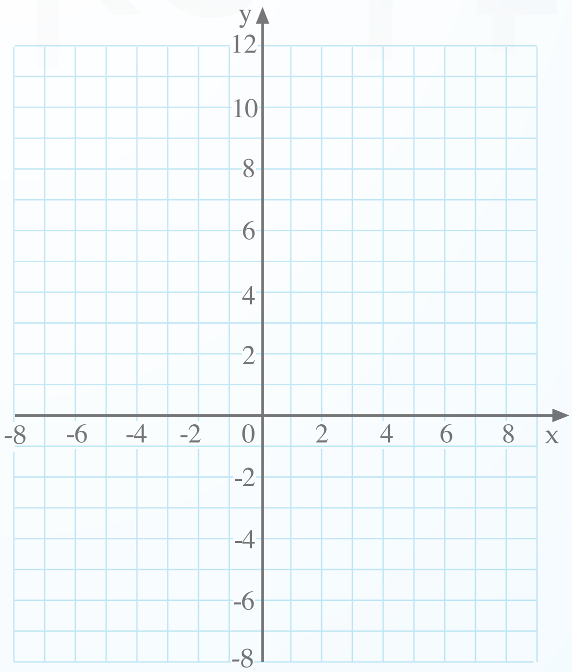
20. y = 4x – 1

	x	y
Point 1		
Point 2		
Check		



21. y = x

	x	y
Point 1		
Point 2		
Check		



Example

Given the gradient and y intercept, draw and label each of the following lines.

a) gradient 4, y intercept 2.

b) gradient $\frac{2}{5}$, y intercept $^-1$.

c) gradient $\frac{^-3}{4}$, y intercept $^-5$.

d) gradient $^-0.5$, y intercept 6.

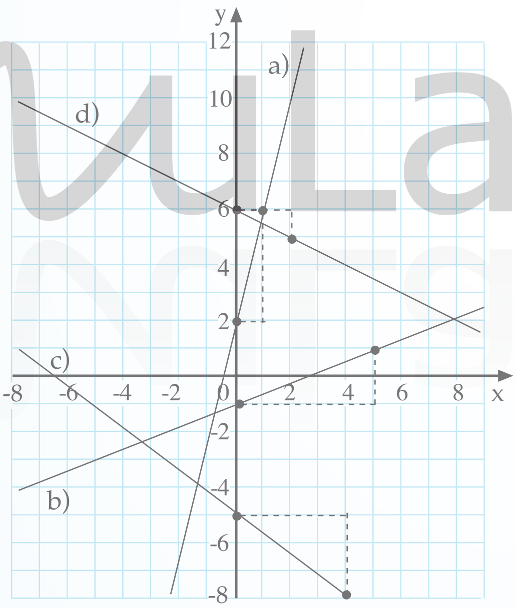
Solution

a) gradient $\frac{4}{1}$ (as a fraction), y intercept 2.

b) gradient $\frac{2}{5}$, y intercept $^-1$.

c) gradient $\frac{^-3}{4}$, y intercept $^-5$.

d) gradient $\frac{^-1}{2}$ (as a fraction), y intercept 6.



Extra Examples

Example

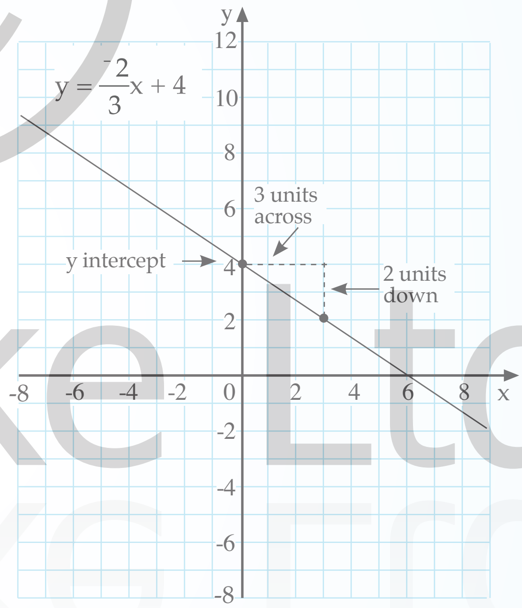
Draw the graph of $y = \frac{^-2}{3}x + 4$ using the gradient-intercept method.

Solution

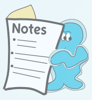
Begin by marking in the y intercept (4) and then from this point move 3 units across and 2 units down which represents a gradient of $\frac{^-2}{3}$.

Draw a line from your position through the marked y intercept.

The line drawn is $y = \frac{^-2}{3}x + 4$.



2.9 Parabolas



Plotting Points

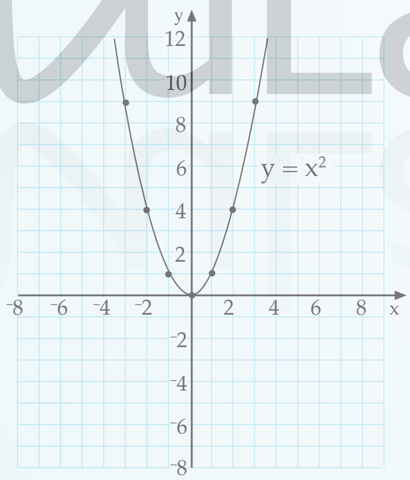
A quadratic is an expression where the highest power of x is x^2, e.g. $y = x^2$, $y = x^2 + 2$, $y = x^2 + 4x$, $y = x^2 - 2x + 1$ etc.

The graph of a quadratic is a parabola. The simplest parabola is that of $y = x^2$. We can draw a parabola by first plotting a series of points and then joining them with a smooth curve.

Consider the quadratic $y = x^2$. We complete a table of values for the quadratic, by squaring each x value.

x	$y = x^2$	
$^-3$	9	← $y = {^-3}^2 = 9$
$^-2$	4	← $y = {^-2}^2 = 4$
$^-1$	1	← $y = {^-1}^2 = 1$
0	0	← $y = 0^2 = 0$
1	1	← $y = 1^2 = 1$
2	4	← $y = 2^2 = 4$
3	9	← $y = 3^2 = 9$

We then plot these points on a graph and join them with a smooth curve.

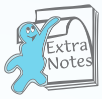


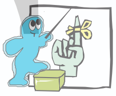
The base or vertex of the parabola $y = x^2$ is the point (0, 0). From this point we move out 1 up 1 to the next point, out 1 up 3 to the next point, out 1 up 5 to the next point, out 1 up 7 etc.

See the 'Odd Number Pattern' on the right.

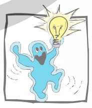
Similarly we can move left and get the same pattern.

Note also that a parabola is symmetrical about a vertical line drawn through the base or vertex of the parabola. The vertical line is mirror line for the plotted points.





On most calculators if you directly enter $^-2^2$, you will get the incorrect answer $^-4$, as the calculator squares just the 2 and leaves the answer negative. You must use brackets to square a negative number if you are to get the correct positive result, e.g. to square $^-2^2$, we enter $(^-2)^2$.

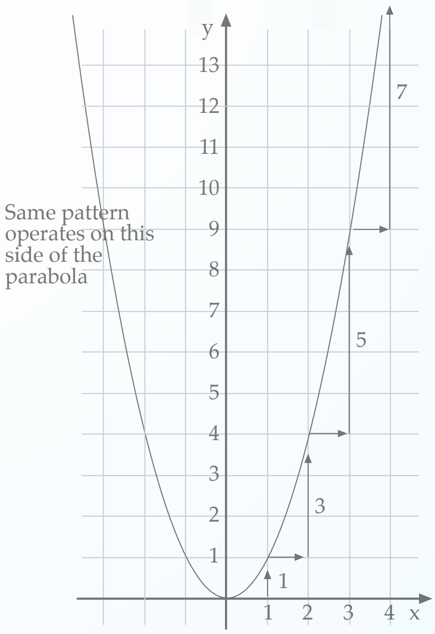


Odd Number Pattern

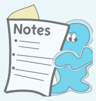
Once we know where the base (turning point) of a parabola is, we can use the pattern of odd numbers:

Out 1 up 1,
then out 1 up 3,
then out 1 up 5,
then out 1 up 7 etc.

to quickly plot points as shown in the parabola on the left. This 'Odd Number Pattern' holds for any parabola with a single x^2 in its equation.



2.12 Factorising



Factorising

Factorising is the reverse procedure of expanding. When we factorise we rewrite the expression as a product of factors.

We adopt different factorising techniques for different types of expressions.

Expanding

$(x + 1)(x - 4) \quad = \quad x^2 - 3x - 4$

Factorising

Common Factors

We look at each term of the expression and identify the highest common factor i.e. the largest term that will divide into each element of the expression. We then put this on the outside of the brackets and divide it into each term.

Consider 12ab + 8b

The highest common factor of 12ab and 8b is 4b as 4 is the highest factor of 12 and 8 and b is the only common factor of ab and b. Therefore we write

$$12ab + 8b = 4b(3a + 2)$$

Quadratics

Sometimes we cannot find a common factor for an expression or even for groups within an expression. Consider an expression of the form $\mathbf{ax^2 + bx + c}$, where $a \neq 0$.

We have already graphed expressions of this type which are called **quadratics** and later in this section we look to solve this type of equation. An essential part of solving quadratics is the ability to factorise them.

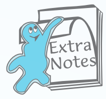
To factorise quadratics which have an 'a' term equal to 1, we attempt to find two numbers that multiply to give the 'c' term and add to give the 'b' term.

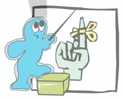
Consider $\quad x^2 + 5x + 6$

Two numbers that multiply to give 6 and add to give 5 are 2 and 3.

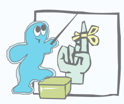
Therefore we write $(x + 2)(x + 3)$ which are the factors of the quadratic (the order of the brackets does not matter, i.e. $(x + 2)(x + 3) = (x + 3)(x + 2)$.

If we were to expand these factors, we would obtain $x^2 + 5x + 6$, the quadratic expression we started with.





A 'factor' is a term that divides into an expression or number without remainder.



After finding your two numbers always check that they multiply to give the 'c' term and add to give the 'b' term.

2.15 Change of Subject

Change of Subject

The subject of the equation $v = u + at$ is 'v'.

Sometimes to make calculations simpler it may be necessary to make one of the other variables in the equation the subject. For example we may know the circumference of a circle but need to find the radius. To do this we rearrange the formula for the circumference of a circle so r is the subject (i.e. instead of $C = 2\pi r$, we get $r = \frac{C}{2\pi}$.

To change the subject of an equation we apply the same techniques we used when solving equations.

Consider again the equation $v = u + at$. To make 't' the subject of the equation we isolate 't' on one side of the equation by applying the same rule we used when solving equations, 'whatever you do to one side of an equation you must do to the other'. This ensures the equation stays in balance.

v	$= u + at$	Equation with v the subject
$u + at$	$= v$	Swap around to get t on the left
$u + at - u$	$= v - u$	Subtract u from both sides
at	$= v - u$	Simplify
$\frac{at}{a}$	$= \frac{v-u}{a}$	Divide both sides by a
t	$= \frac{v-u}{a}$	Same equation with t the subject

Extra Notes

When we change the subject of an equation, the equation is still the same except a different variable has been made the subject.

The equation $x = y + 4$ is the same as $y = x - 4$ except x is the subject of the first equation and y the subject of the second equation.

In the first equation when $y = 1$, $x = 5$.

In the second equation if $y = 1$ then x also equals 5.

Example

Make y the subject of the equation

$$x + 2y = 3$$

Solution

$x + 2y$	$= 3$	
$x + 2y - x$	$= 3 - x$	Subtract x from both sides
$2y$	$= 3 - x$	Simplify
$\frac{2y}{2}$	$= \frac{3-x}{2}$	Divide both sides by 2
y	$= \frac{3-x}{2}$	Simplify

Extra Examples

3.0 Measurement

3.1 Metric Units

Length

The metric system was developed in the late 1700s to standardise units of measurement in Europe.

New Zealand adopted the metric system in 1969.

In the metric system each type of measurement has a base unit to which prefixes are added to indicate multiples of ten.

The base unit for length is metres. If you wish to convert from a smaller unit to a larger unit (e.g. mm to m) move the decimal point to the left in the number you are converting. This is the same as dividing by the appropriate power of 10.

If you wish to convert from a larger unit to a smaller unit (e.g. m to mm) move the decimal point to the right in the number you are converting. This is the same as multiplying by the appropriate power of 10.

Study the table below. Note that some of the prefixes have been highlighted in blue, these are the ones that are not commonly used.

Prefix	Value
kilo	1000
hecto	100
deca	10
base unit (metres)	1
deci	0.1
centi	0.01
milli	0.001

From the table it is evident that

10 mm = 1 cm

100 cm = 1 m

1000 mm = 1 m

1000 m = 1 km

The common abbreviations we use for length in the metric system are:

m for metre
mm for millimetre
cm for centimetre
km for kilometre

To multiply or divide on a calculator by a power of ten use the EXP or 10^n button.

Example

Change the following to the unit indicated in brackets.

a) 35 mm (cm)
b) 950 m (km)
c) 2.65 m (cm)
d) 53.4 cm (mm)

a) Since there are 10 mm in a cm and we are going from a smaller to a larger unit we divide by 10.

35 mm ÷ 10 = 3.5 cm

b) Since there are 1000 m in a km and we are going from a smaller to a larger unit we divide by 1000.

950 m ÷ 1000 = 0.95 km

c) Since there are 100 cm in a m and we are going from a larger to a smaller unit we multiply by 100.

2.65 m x 100 = 265 cm

d) Since there are 10 mm in a cm and we are going from a larger to a smaller unit we multiply by 10.

53.4 cm x 10 = 534 mm

Achievement – Use the scales on the next page to answer the following questions.

126. What is 13 inches in centimetres?

127. What is 94 centimetres in inches?

128. What is 25 inches in centimetres?

129. What is 85 centimetres in inches?

130. What is 200 metres in yards?

131. What is 600 yards in metres?

132. What is 1500 metres in yards?

133. What is 1400 yards in metres?

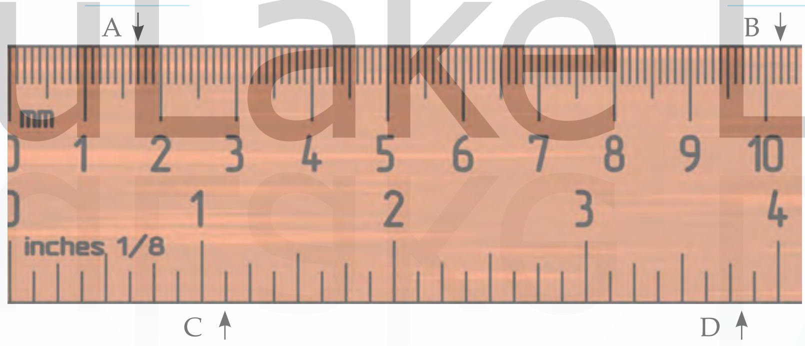
134. What is 135 km in miles?

135. What is 57 miles in km?

136. What is 115 km in miles?

137. What is 29 miles in km?

Achievement – Use the ruler below to answer the following questions.



138. What is 70 mm in inches?

139. What is $3\frac{5}{8}$ inches in millimetres?

140. What is 57 mm in inches?

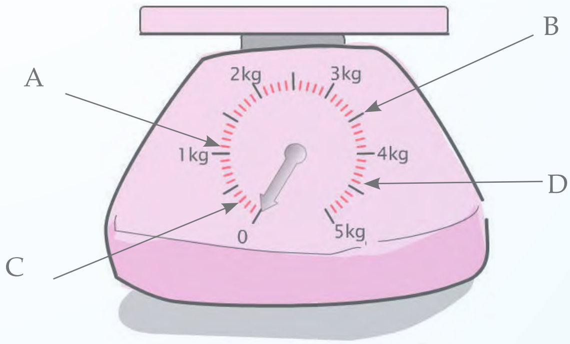
141. What is $2\frac{7}{8}$ inches in millimetres?

142. Label the four marked points, A, B, C and D on the ruler above.

143. a) What value do the unmarked divisions have on the scales on the right?

b) Read the marked weights on the scales.

A = B = C = D =



Merit – For each question below draw a diagram (if necessary) and then solve.

228. An isosceles triangle has a perimeter of 95 cm. The base length of the triangle is 39 cm. What are the other two lengths?

229. A square garden has an area of 30.25 m^2. What is its perimeter?

230. The cross-sectional area of the shaft of a key is a quarter-circle. If the radius of the quarter-circle is 3.4 mm, find the area and perimeter.

231. The area of a trapezium is 84 m^2. If the height is 12 metres and the parallel sides differ by 2, find the length of the two parallel sides.

232. The area of a parallelogram is 363 cm^2. If the height of the parallelogram is 16.5 cm what is the length of the base?

233. The area of a triangular mat is 10.5 m^2 and its base length is 7 m. What is its height?

234. A dog is tied to a point in the centre of a backyard. The length of the rope is 12 metres. What is the area of the region the dog can explore?

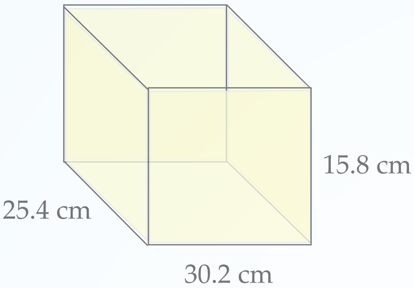
235. The area of a triangle is 104 m^2. The base and height of the triangle differ in length by 3 metres. Find the base and height lengths of the triangle.

236. A rectangular shaped room has width, half its length. If the area of the room is 18 m^2, what is the width and length?

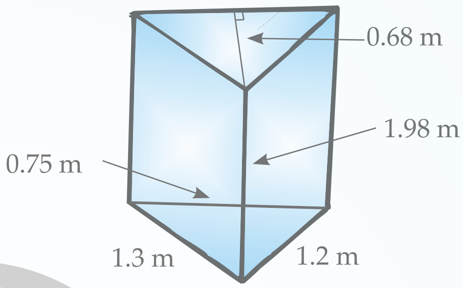
237. A large pizza has a diameter of 30 cm and a small pizza a diameter of 15 cm. How many small pizzas do you need to buy to get the same amount as a large pizza?

Merit – Find the surface area of the following. Sketch a net in the space provided if required.

273.

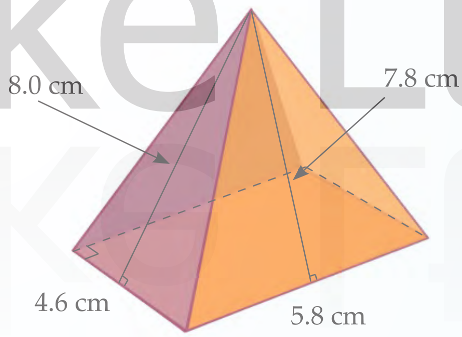


274.



275.

276.



343. Study the distance chart below for Australia's Southwest and answer the questions related to it.

Australia's South West Distance Chart

	Perth	Bunbury	Collie	Busselton	Dunsborough	Bridgetown	Margaret River	Manjimup	Pemberton	Albany	Denmark
Bunbury	179 / 2:30										
Collie	205 / 3:00	56 / 0:50									
Busselton	227 / 3:15	53 / 0:55	113 / 1:30								
Dunsborough	252 / 3:45	78 / 1:30	139 / 2:00	23 / 0:30							
Bridgetown	260 / 3:30	93 / 1:15	96 / 1:15	106 / 1:30	131 / 2:00						
Margaret River	273 / 4:00	102 / 1:45	159 / 2:15	48 / 0:45	46 / 0:40	149 / 2:00					
Manjimup	296 / 4:00	130 / 1:45	133 / 1:45	118 / 1:30	143 / 2:00	35 / 0:30	157 / 2:00				
Pemberton	328 / 4:30	161 / 2:15	164 / 2:15	137 / 1:45	162 / 2:15	67 / 1:00	138 / 1:45	31 / 0:30			
Albany	416 / 5:00	349 / 4:30	294 / 3:45	329 / 4:15	355 / 5:00	247 / 3:15	376 / 4:45	211 / 2:45	238 / 3:00		
Denmark	420 / 5:00	314 / 4:00	286 / 3:45	301 / 3:45	327 / 4:15	220 / 2:30	324 / 4:00	184 / 2:15	185 / 2:15	53 / 0:45	
Walpole	415 / 5:30	248 / 3:15	251 / 3:15	235 / 3:00	261 / 3:30	154 / 2:00	258 / 3:15	118 / 1:30	119 / 1:30	118 / 1:30	65 / 0:45

Distance (kilometres)
Time (hours)

To convert from km to miles multiply by 0.621

a) What is the distance in kilometres from Perth to Margaret River?

b) What is the distance in kilometres from Bunbury to Pemberton?

c) How long does it take to travel from Collie to Albany?

d) What speed in kilometres per hour are you travelling at to get from Perth to Walpole in the designated time?

e) What is the distance in kilometres from Perth to Albany via Bridgetown?

f) How long does it take to travel from Denmark to Collie via Manjimup?

g) The distance from Busselton to Manjimup is the same as the distance from Walpole to what town?

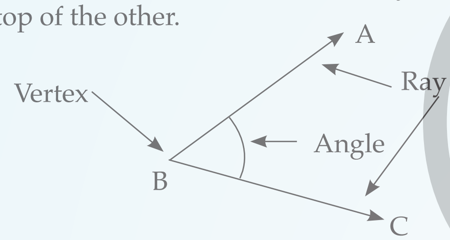
h) The distance from Perth to Busselton is 100 km less than the distance from Denmark and what town?

4.0 Geometry

4.1 Angle Revision

Angles

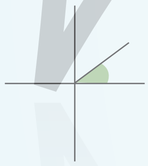
An angle is a measure of turn and is measured in degrees. An angle is formed from the intersection of two rays. The size or measure of an angle is the amount of turn needed to take one ray and place it on top of the other.



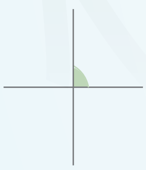
We represent or name an angle by using three letters. The middle letter is always the vertex of the angle. The angle above could be labelled as angle ABC or angle CBA.

Sometimes instead of the word angle we use the symbol ∠. So angle ABC is the same as ∠ABC.

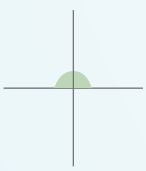
Some angles or group of angles are given specific names. A summary of these are:



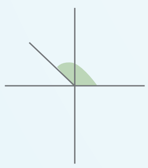
An acute angle is an angle between 0° and 90°.



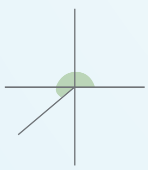
A right angle is an angle of 90°.



A straight angle is an angle of 180°.



An obtuse angle is an angle greater than 90° and less than 180°.



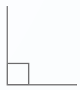
A reflex angle is an angle greater than 180° and less than 360°.

Extra Notes

The symbol for degrees is °. There are 360° in a complete turn.

We can also name an angle by using just a single letter. The angle on the left could be written as ∠B. The single letter we use is always the vertex of the angle.

A right angle is often represented by a small square.



Merit – Answer the following.

43. Give the sum of the interior angles of a 12-sided polygon.

44. Give the measure of an interior angle of a regular polygon with nine sides?

45. The measure of each interior angle of a regular polygon is 144°. How many sides has the polygon?

46. The measure of each interior angle of a regular polygon is 120°. How many sides has the polygon?

47. A regular polygon has 20 sides. What is the measure of each exterior angle?

48. A regular polygon has 15 sides. What is the measure of each interior angle?

49. A regular polygon has an exterior angle measure of 18°. How many sides has the polygon?

50. A regular polygon has an exterior angle measure of 45°. How many sides has the polygon?

Excellence – Answer the following.

51. Explain why it is not possible to have a regular polygon with an exterior angle of 28°.

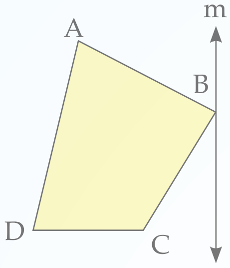
52. Explain why it is not possible to have a regular polygon with an interior angle of 134°.

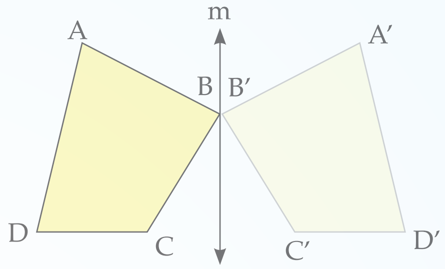
53. Each exterior angle of a regular polygon measures 15°.
    a) How many sides does the polygon have?
    b) What is the size of each interior angle of the polygon?
    c) What is the sum of the exterior angles of the polygon?

54. Each exterior angle of a regular polygon measures 12°.
    a) How many sides does the polygon have?
    b) What is the size of each interior angle of the polygon?
    c) What is the sum of the exterior angles of the polygon?

Example

Reflect the following shape in the mirror line m and label appropriately. Which point(s) remain invariant after the reflection?

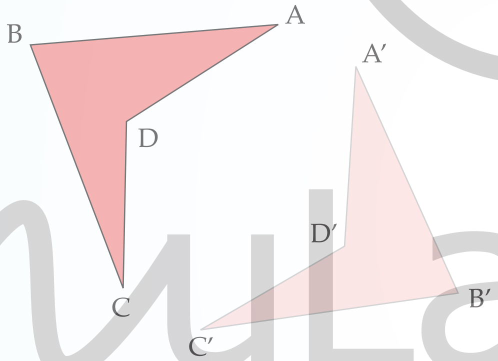


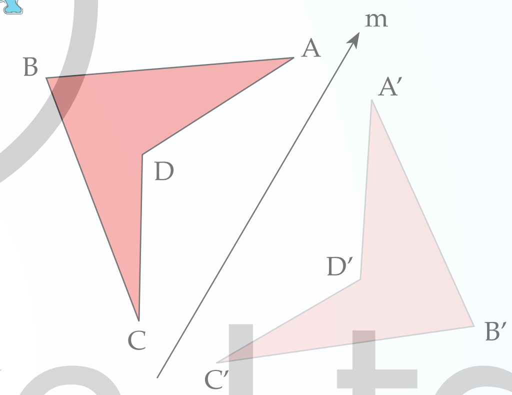


The point which remains invariant under the reflection are those on the mirror line, i.e. the point B.

Example

The figure A′B′C′D′ is the image of ABCD under reflection. Draw in the mirror line m.





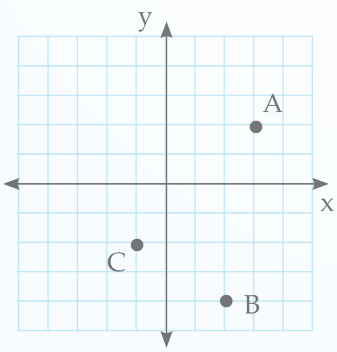
Example

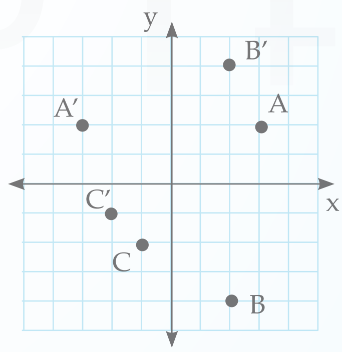
Using the grid provided plot the image of the given points after reflection in the mirror line described.

a) A(3, 2) reflected in the y axis.

b) B(2, $^{-}4$) reflected in the x axis.

c) C($^{-}1$, $^{-}2$) reflected in the line y = x.





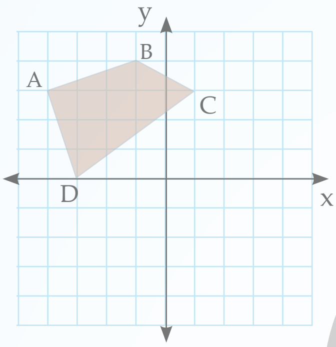
a) A′($^{-}3$, 2)

b) B′(2, 4)

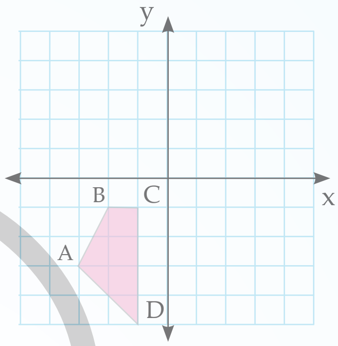
c) C′($^{-}2$, $^{-}1$)

Achievement – Translate the following as described.

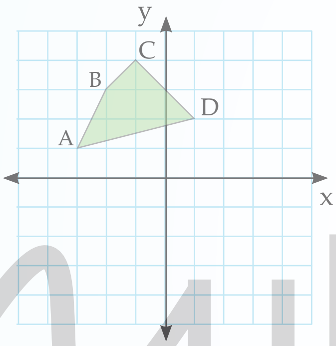
159. Translate by 3 units to the right and 2 units down.



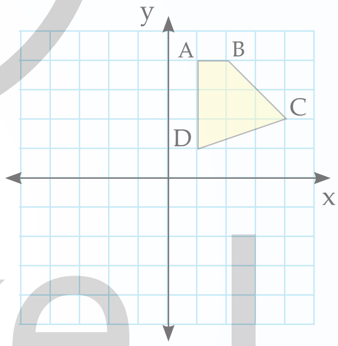
160. Translate by 2 units to the left and 5 units up.



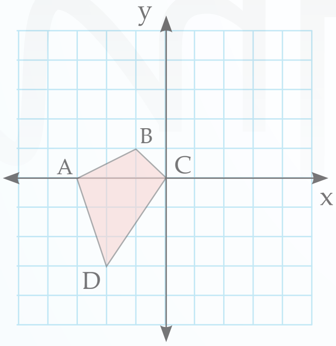
161. Translate by 3 units to the right and 1 unit up.



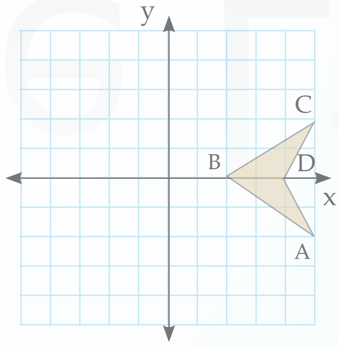
162. Translate by 5 units to the left and 4 units down.



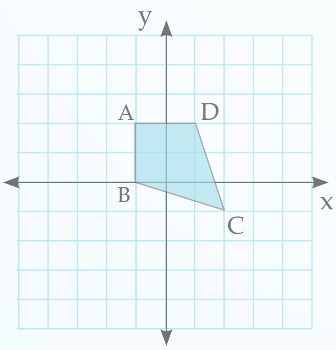
163. Translate ABCD by the vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.



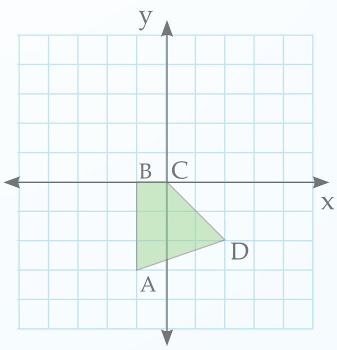
164. Translate ABCD by the vector $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$.



165. Translate ABCD by the vector $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$.



166. Translate ABCD by the vector $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$.



Trigonometry cont...

In summary the sine function enables us to find the length of the opposite side, of a right-angled triangle, relative to a given angle if we know the length of the hypotenuse.

The cosine function enables us to find the length of the adjacent side, of a right-angled triangle, relative to a given angle if we know the length of the hypotenuse.

The tangent function enables us to find the length of the opposite side of a right-angled triangle relative to a given angle when we know the length of the adjacent side.

It is important that you remember these ratios.
The mnemonic SOHCAHTOA (so-car-toe-a) should help you.

SOH

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

CAH

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$

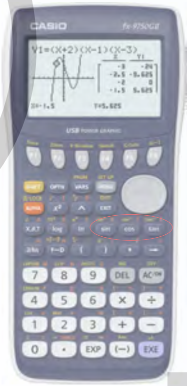
TOA

$$\tan A = \frac{\text{opp}}{\text{adj}}$$

Extra Notes

Extra Notes

Locate the sin, cos and tan buttons on your calculator now and remember that your calculator needs to be set to degrees when using these three functions.



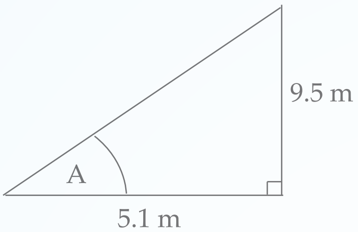
To set the Casio 9750GII to degrees press SHIFT SETUP scroll down to Angle and press F1 to change to degrees.

To set the TI 30XB to degrees press mode and then select DEG using ►. Press 2nd mode to quit.

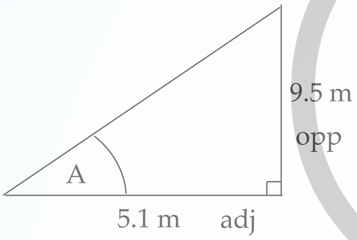
To set the Casio fx-82MS to degrees press MODE MODE 1 .

Example

Find angle A of the right-angled triangle using trigonometry.



We begin by labelling the two sides of the right-angled triangle that are given to us.



We identify which trigonometric ratio involves the two sides opp and adj.

It is the tan ratio, i.e. $\tan A = \dfrac{\text{opp}}{\text{adj}}$.

We substitute for both the opp and the adj in the trigonometric ratio.

$\tan A = \dfrac{\text{opp}}{\text{adj}}$

$\tan A = \dfrac{9.5}{5.1}$ opp = 9.5 and adj = 5.1

$\tan A = 1.8627...$ simplifying $\dfrac{9.5}{5.1}$

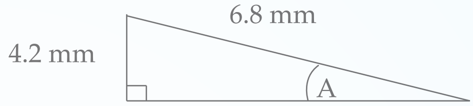
$A = \tan^{-1}(1.863)$ $\tan^{-1}$ gives us the angle A

$A = 61.8°$ (1 dp) using your calculator

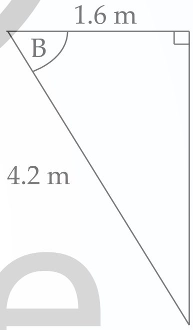
Make sure your calculator is set to degrees when you find $\tan^{-1}(1.863)$.

Achievement – Find the marked angle(s) of the right-angled triangles by using trigonometry. Round your answer to 1 dp.

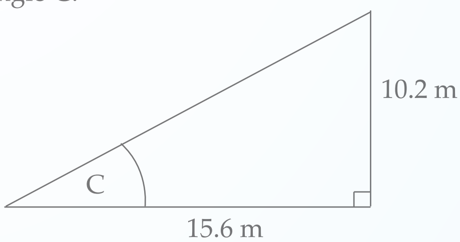
230. Find angle A.



231. Find angle B.



232. Find angle C.



4.8 Compass Directions and Bearings

Compass Directions and Bearings

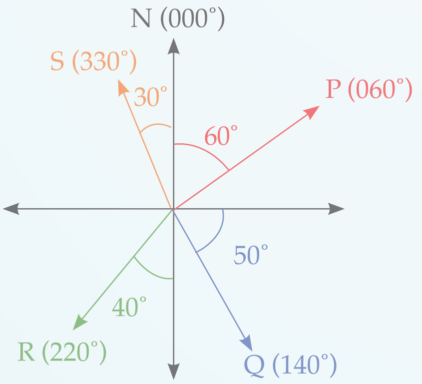
A compass is used to find a direction or bearing.

The diagram of the compass below shows the four main directions North, South, East and West, known as the cardinal points as well as the points halfway between each of the cardinal points, namely North East (NE), South East (SE), South West (SW) and North West (NW).

Bearings are used to represent the direction of one point relative to another and are always represented as a three digit number in degrees.

A true bearing to a point is the angle from North measured in degrees in a clockwise direction to the required point. True bearings are usually just referred to as bearings.

Consider the point P below. Its bearing is 060° because it is 60° in a clockwise direction from North (000°)



Similarly Q would have a bearing of 140° (90° + 50° in a clockwise direction from N) and R a bearing of 220° (180° + 40° in a clockwise direction from N) and S a bearing of 330° (360° – 30° in a clockwise direction from N).

Extra Notes

Familiarise yourself with the compass below and the cardinal points as well as the other commonly labelled divisions.

There are 360° in a complete turn that's why bearings range from 000° to 360°.

5.0 Statistics

5.1 Introduction

Statistical Enquiry Cycle

At Year 10 students need to be able to plan and conduct an investigation using the Statistical Enquiry Cycle.

The cycle involves five stages as depicted below.

We usually undertake an investigation by following the Statistical Enquiry Cycle in sequence but other times, after completing a stage, a rethink may be required and it may be necessary to take a step back.

Often completing an investigation identifies other factors that could be investigated, which means the cycle can start over.

Extra Notes

Pose a Question

This is where you decide what you would like more information about. A good idea is to start with the phrase

"I wonder if ..."

The 'problem' helps you identify what information you need to know in order to answer the question.

Plan a survey or experiment

You need to know what you will measure and how you will do it.

- What data do you need?
- How will you collect the data in order to answer the problem?
- What information will you need to record?
- How will you record the information?

Gather the Data

This is where your data is collected, managed and organised. It should be organised in such a way that it is easy to extract information from (often as a table). You may need to clean the data.

Analyse the Data

This is where you look at the data to see what it tells you about your problem.

Graph the data in a way that best answers the problem/question. Collect statistics (measures of the middle and spread) and extremes of your data so that you can present evidence to support any conclusion.

Form Conclusions

What did you learn from the investigation?

- What do the graphs tell you?
- What differences are there in the statistics?
- Can you infer that any difference in the sample is also in the population?
- Does the investigation suggest new problems that could be investigated?

Achievement /Merit – Calculate both quartiles, interquartile range and range for the following. You may use your calculator.

6. 13, 35, 46, 57, 35, 57, 44, 57, 68, 37, 26, 35, 77, 24, 46, 57, 13, 54, 66, 35, 25, 35, 79, 24

7. 3.35, 7.22, 5.57, 4.55, 5.64, 4.51, 7.33, 5.57, 4.24, 7.34, 5.22, 4.74, 5.81, 5.64, 4.45, 8.11

8. 59, 30, 41, 67, 23, 35, 91, 14, 34, 89, 34, 56, 56, 52, 43, 30, 32, 43, 56, 31, 57, 74, 69

9. 14.5, 10.6, 11.7, 7.4, 10.3, 9.9, 12.2, 5.2, 8.5, 6.7, 8.5, 10.1, 9.2, 8.5, 10.4

10. 124, 451, 903, 774, 182, 390, 316, 854, 503, 959, 731, 175, 417, 380, 102, 355, 837, 215, 161

11. 4.9, 8.7, 3.8, 1.6, 1.0, 5.9, 6.5, 2.9, 3.0, 7.6, 6.3, 0.2, 5.4, 3.5, 8.4, 9.4, 11.6, 7.2

12. Anthea's swim meet times (seconds) for 50 metres freestyle over the season are:

36.4, 33.9, 30.4, 35.3, 32.7, 30.2, 35.3, 31.8, 34.9, 31.5, 32.3, 31.0, 33.6, 34.4, 31.3, 30.8, 31.7, 34.2

Calculate her median, quartiles, interquartile range and range times and use them to describe the data.

13. The number of days absent, in a term, for a Year 10 class of 30 students is:

0, 6, 3, 8, 4, 15, 2, 4, 9, 12, 3, 32, 2, 1, 5, 4, 0, 4, 2, 1, 0, 2, 0, 10, 1, 1, 6, 4, 1, 2

Calculate the median, quartile, interquartile range and range of absences and use them to describe the data.

14. Tony has played a total of 30 games of cricket through the season, fifteen 40 over games and fifteen twenty20 games. The runs he scored for each game are given below. Compare his performance in both forms of the game, for the season, by calculating the medians, LQs, UQs, ranges and IQRs.

40 over: 58, 31, 6, 2, 9, 27, 18, 24, 25, 27, 34, 27, 31, 28, 30.

Twenty20: 15, 1, 22, 54, 47, 36, 22, 34, 19, 22, 40, 38, 29, 41, 35.

15. Two groups were given the same test marked out of 20. One in school and the other for homework. Find the median, quartiles and range of both groups and compare.

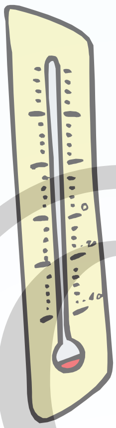
Home: 12, 17, 19, 20, 13, 14, 15, 15, 15, 12, 16, 18, 14, 15, 13, 17, 18, 14, 16, 14, 16, 15, 13, 15, 17

School: 20, 15, 18, 13, 12, 5, 11, 11, 19, 13, 16, 14, 7, 18, 9, 10, 12, 12, 13, 14, 13, 17, 16, 15, 13

21. The daily high temperature (˚C) for one year in a European city are represented in the frequency table below.

a) Complete the midpoint column.

Temp (˚C)	Midpoint (x)	Frequency Days - (f)
0 – < 2.5		4
2.5 – < 5.0		28
5.0 – < 7.5		33
7.5 – < 10.0		35
10.0 – < 12.5		27
12.5 – < 15.0		50
15.0 – < 17.5		42
17.5 – < 20.0		28
20.0 – < 22.5		44
22.5 – < 25.0		74
	TOTAL	365



b) What proportion of the days in the year was the daily high temperature in the 7.5 to 10.0 ˚C range?

c) What percentage of the days in the year was the daily high temperature in excess of 20.0 ˚C?

d) What fraction of the year was the daily high temperature below 5.0 ˚C?

e) Using your calculator find the mean, mode, median, LQ, UQ and IQR for the daily high temperatures.

f) Summarise the expected daily high temperatures of the European city using the applicable statistics from part d).

22. The weekly wage of employees in a company are given in the frequency table below.

Wage ($)	Frequency Days - (f)
550 – < 600	40
600 – < 650	34
650 – < 700	21
700 – < 750	18
750 – < 800	12
800 – < 850	10
850 – < 900	8
900 – < 950	6
950 – < 1000	5
1000 – < 1050	4
1050 – > 1100	2
1100 – < 1150	1

a) What is a better measure of the 'average' wage in the company, the mean or the median? Calculate both and justify your answer.

b) Calculate the IQR for the data and explain what it means in the context of the data.

Box and Whisker Plot

Box and Whisker Plot

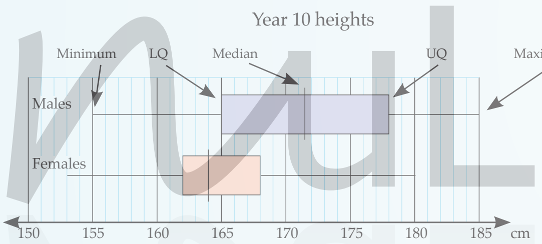
A box and whisker plot comprises a rectangle (box) which displays the lower and upper quartiles. A line through the rectangle represents the median, while two lines drawn from the rectangle (whiskers) extend to the minimum and maximum values. Extreme values that are above (or below) the rest of the data are shown as outliers (marked with an x).

A box and whisker plot is an efficient way of handling a large amount of data and conveys an excellent feeling for the distribution of the data.

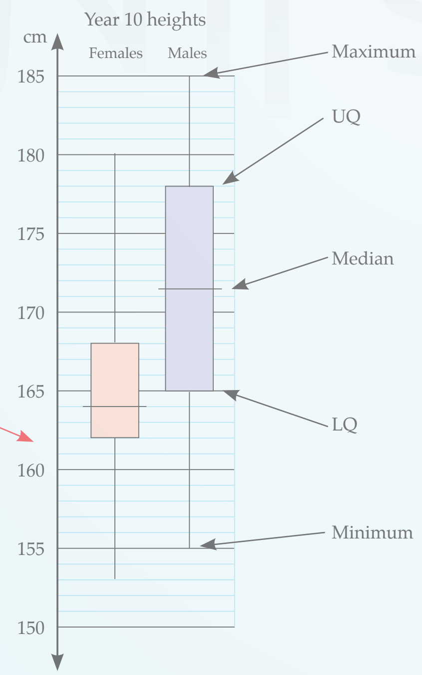
Side-by-side box and whisker plots make it easy to compare two groups by looking for similarities and differences.

One disadvantage of a box and whisker plot is that the exact values and details of the original data are not retained. A box and whisker plot can be drawn either horizontally or vertically (see below).

Box and whisker plots are also particularly useful in the analysis phase of the Statistical Enquiry Cycle especially if we have posed a comparison type question.



Box and whisker plot drawn horizontally. Minimum to the left, maximum to the right.



Box and whisker plot drawn vertically. Maximum at the top, minimum at the bottom.

Extra Notes

Box and whisker plots are particularly useful in the analysis phase of the Statistical Enquiry Cycle especially if you have posed a comparison type question.

A box and whisker plot is right-skewed if most of the observations are on the left hand side of the distribution.

Conversely a box and whisker plot is left-skewed if most observations are on the right hand side of the distribution.

If the box and whisker plot is symmetrical the values will be evenly divided at the median.

When comparing two box and whisker plots look for:

Differences or similarities in the median.

Differences in the widths of the box (IQR).

Differences from whisker to whisker (range).

Presence of outliers (extreme values).

Overlap or separation between parts of the two box and whisker plots.

Symmetry or skewness.

Example (same data as on page 321)

Draw a back-to-back stem and leaf plot of the height (cm) of a sample of 30 male and female Year 10 students. *(Source censusatschool)*

female	170	male	171
female	157	male	178
female	161	male	167
female	174	male	157
female	162	male	164
female	168	male	170
female	166	male	169
female	165	male	179
female	160	male	180
female	163	male	174
female	163	male	165
female	162	male	169
female	165	male	172
female	153	male	170
female	171	male	185
female	164	male	172
female	155	male	181
female	162	male	156
female	172	male	178
female	164	male	174
female	164	male	180
female	158	male	178
female	166	male	160
female	164	male	155
female	163	male	181
female	168	male	180
female	165	male	172
female	178	male	165
female	180	male	161
female	158	male	168

Comment on the back-to-back stem and leaf plot using such terms as peaks, clusters, outliers, skewness etc. Quote applicable statistics.

We begin by identifying the stem for our stem and leaf plot. As the heights range from 153 to 185 we could use 4 branches, namely 15, 16, 17 and 18. Alternatively we could use 8 branches by splitting up each branch of '10' into two branches of '5' each.

Heights from 150 cm to 154 cm would go into the first branch 15, while 155 cm to 159 cm would go into the second branch of 15 (see below). Once we have organised our stem then we enter the data directly into the stem and leaf plot from the table above.

Heights (unordered)

Females	Stem	Males
3	15	
8 8 5 7	15	7 6 5
3 4 4 4 2 4 2 3 3 0 2 1	16	4 0 1
5 8 6 5 5 6 8	16	7 9 5 9 5 8
2 1 4 0	17	1 0 4 2 0 2 4 2
8	17	8 9 8 8
0	18	0 1 0 1 0
	18	5

cont...

We now go through the process of ordering each branch of our stem and leaf plot from smallest to largest. From the ordered stem and leaf plot we can calculate the median and quartiles.

Heights (ordered)

	Females	Stem	Males	
	3	15		
LQ	8 8 7 5	15	5 6 7	
	4 4 4 4 3 3 3 2 2 2 1 0	16	0 1 4	LQ
Median	8 8 6 6 5 5 5	16	5 5 7 8 9 9	
UQ	4 2 1 0	17	0 0 1 2 2 2 4 4	
	8	17	8 8 8 9	Median
	0	18	0 0 0 1 1	
		18	5	UQ

Either by using your calculator or from the ordered stem and leaf plot, above, we get the following:

	Year 10 students	
	Female	Male
Number	30	30
Median	164	171.5
LQ	162	165
UQ	168	178
IQR	6	13
Minimum	153	155
Maximum	180	185

From the back-to-back stem and leaf plot we identify peaks for 170 to 174 cm for males and 160 to 164 cm for females.

Male and female heights are generally symmetrical about their respective medians and there are no obvious outliers or skewness.

The average height of Year 10 male students is greater than that of Year 10 female students (median of 171.5 cm compared to that of 164 cm).

75% of Year 10 males students are taller than the median height of female Year 10 students.

The interquartile range of Year 10 male students (13 cm) is more than twice the interquartile range of Year 10 female students (6 cm).

The range in heights of the two groups is similar, 27 cm for the female students and 30 cm for the male students.

Generally, male Year 10 students are taller than their female counterparts, but the spread of the middle 50% of the heights (IQR) is significantly greater for males than for that of the females.

Other Graphs

Line Graphs

Line graphs are used to show how data changes over time. Some advantages of line graphs are they

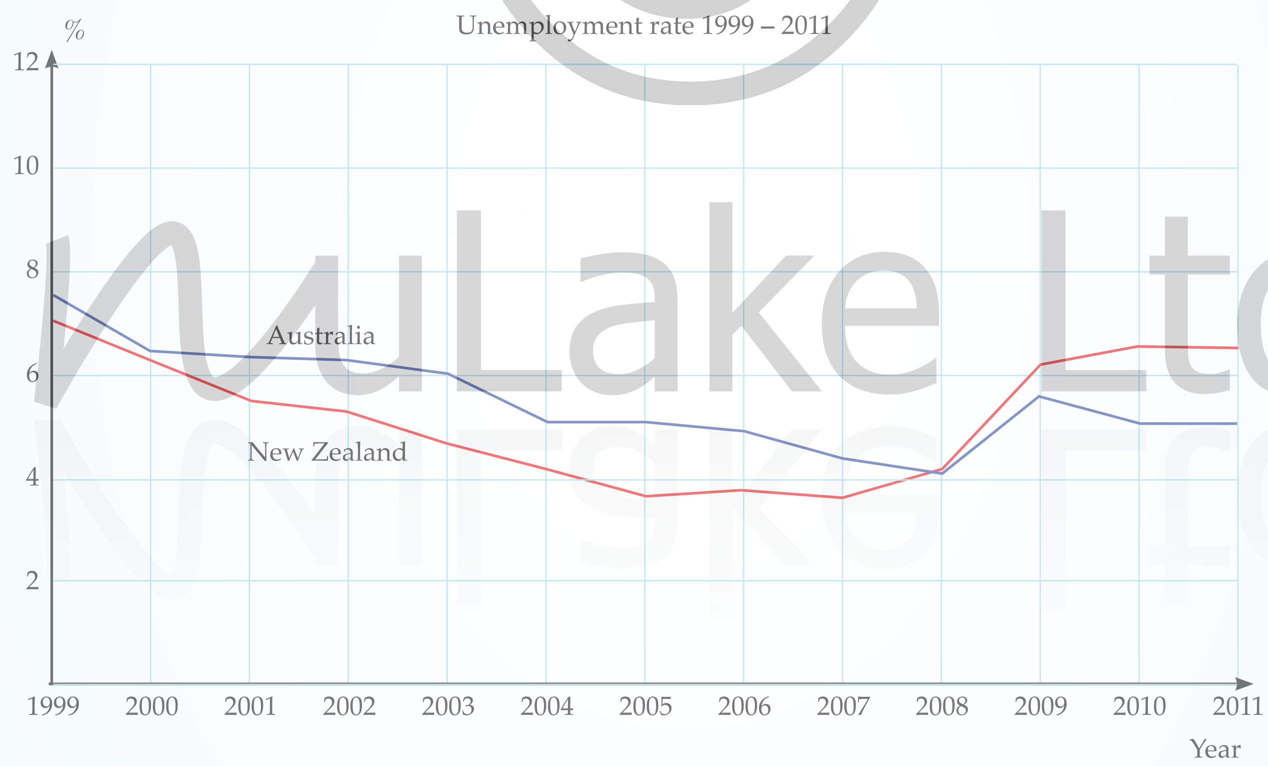
- are easy to read
- show patterns and trends in the data
- enable us to make predictions.

Typical examples of the type of data that can be represented by a line graph include, monthly rainfall, unemployments rates, temperatures etc.

In a line graph the x axis (horizontal) represents the continuous variable, e.g. time, distance etc. and the y axis (vertical) the measurement.

More than one set of data can be displayed on a line graph which is useful when analysing and comparing the tends of two or more data sets.

An example line graph is drawn below. It shows the unemployment rate (%) of Australia and New Zealand over the period 1999 to 2011. *(Source: www.indexmundi.com)*



Pie Charts

A pie chart is based on a circle divided into sectors or slices where each sector represents a proportion or percentage of a whole. Some advantages of a pie chart are they

- can summarise a large data set in a visual form
- require minimal explanation
- are easy to understand because of their widespread use in business and the media etc.

The disadvantages of pie charts are that they can't compare two sets of data, do not show patterns or trends and do not display precise values unless these have been specifically included on the pie chart.

6.0 Probability

6.1 Experimental Probability

Experimental Probability

Probability is the chance or likelihood some event will happen.

Experimental probability is where an experiment is conducted and the relative frequency of an event occurring is calculated from the results of the experiment.

The relative frequency of an event is how often that event occurs divided by all possible outcomes.

Experimental probability is frequently used in research and experiments in social sciences, economics, medicine etc.

For example, to test the effectiveness of an antidote for a particular disease in mice, a number of mice with the disease are injected with the antidote. The experimental probability that a mouse is cured is found by dividing the number of mice cured by the number that were treated with the disease.

If our test was large enough then it is possible to extend the effectiveness of the antidote, i.e. the probability of a mouse being cured, to all mice with the disease.

In order for experimental probability to be meaningful and of value in research, the sample size must be sufficiently large. If we only tested the antidote on five mice with the disease and all five were cured it is not valid to make the claim that the antidote is 100% effective.

If we tested the antidote on 200 mice and 160 were cured then a claim that the antidote was 80% effective against the disease is realistic.

It is important to be familiar with the terms that are used in conjunction with probability. Some of these are listed below.

Sample space is the possible outcomes from an experiment. For the mouse experiment described above the sample space is for a mouse to be cured, or not cured.

A **trial** in the experiment above would be a single mouse injected with the antidote.

An **outcome** is the result of a single trial of an experiment, e.g. was the mouse treated with the antidote cured or not cured.

An **event** is one or more outcomes of a particular experiment, e.g. the event of a mouse being cured in the experiment.

Extra Notes

The relative frequency of an event occurring can be calculated by the formula

$$\frac{\text{Number of times the event occurs}}{\text{The total number of possible outcomes}}$$

Relative frequency is the experimental frequency after a large number of trials.

The experimental probability of an event is an 'estimate' that the event will happen as a consequence of an experiment or from collecting data or by direct observations or experiments.

6.2 Theoretical Probability

Theoretical Probability

The theoretical probability of an event occurring is calculated by identifying all the possible outcomes and determining how likely the required outcome is to occur. Theoretical probability is determined through reasoning or calculation.

To find the theoretical probability of an event we use the ratio

$$P(\text{Event}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

The probability of an event, E occurring can be written as P(E) and will always be a value in the range 0 to 1. An event with a probability of 0 is an impossibility and an event with a probability of 1 is a certainty.

Consider a box containing 5 red marbles. If a single marble is drawn from the box and E is the event a red marble is drawn, then P(E) = 1 (i.e. a certainty).

If E is the event a green marble is drawn, then P(E) = 0 (i.e. an impossibility).

When calculating the probability of an event it is important that we begin by identifying the sample space. The sample space is the set of all possible outcomes. The size or number of elements that make up the sample space is what we need to know.

Consider the throw of a die. The sample space is 1, 2, 3, 4, 5 and 6. So the total number of possible outcomes is 6. If we were to toss a coin the sample space is H, T and the total number of possible outcomes is 2.

The number of favourable outcomes of an event is the number of successful or required results.

The probability that the number '5' shows up on a die when rolled is $\frac{1}{6}$ because the total number of possible outcomes is six (1, 2, 3, 4, 5, 6) and only one of these outcomes, the '5', is favourable.

Theoretical probability is determined through reasoning or calculation.

Equally likely events are events that have the same theoretical probability (or likelihood) of occurring, e.g. the toss of a fair die.

The theoretical probability of an event occurring can be calculated by using the formula

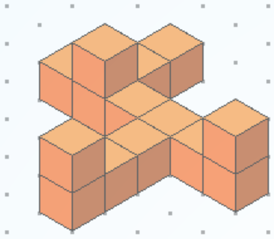
$$\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

The theoretical probability of a certainty is 1.

The theoretical probability of an impossibility is 0.

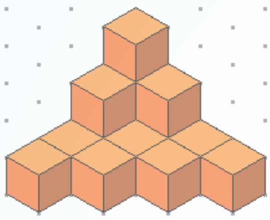
Page 224 cont...

66.



Page 225

67.



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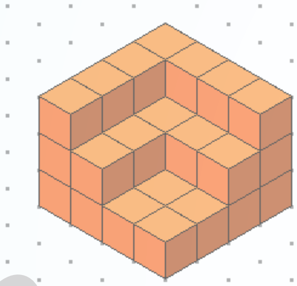

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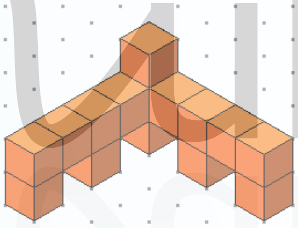

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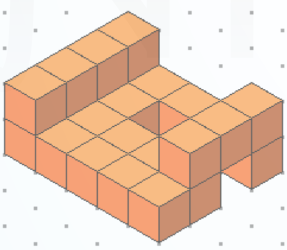

Page 226

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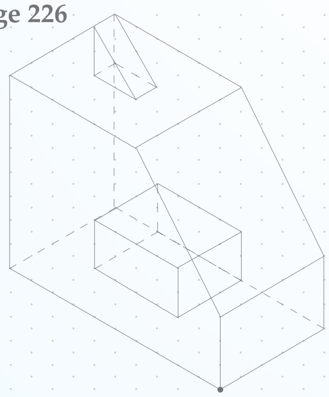


Page 227

72.

Page 229

73.

3	2	2	1	1
2	1			
1				

74.

4	1	
3		
2		
2	1	3

75.

0 or 1 or 2	2		
5	3	2	1
1			1
2			

76.

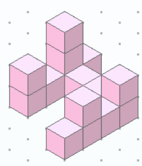
3	2	2	1
2			
2			
1			

77.

5			
2			
3	2	3	1
4			1
1			1

Page 230

78.



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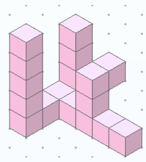

Page 230 cont...

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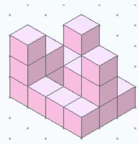

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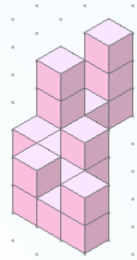

Page 232

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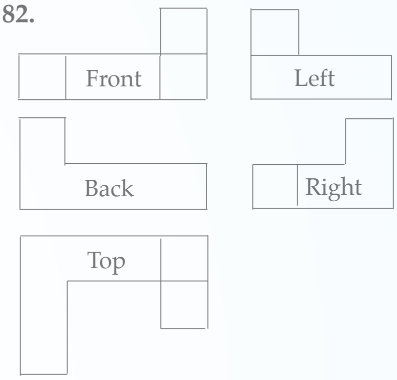

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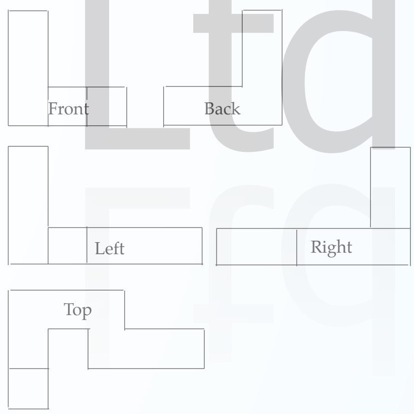

Page 233

84.